Name:

Spring 2019 Math 245 Exam 1

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Writing your name incorrectly will cost you a point. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. even

b. tautology

c. converse

d. predicate

Problem 2. Carefully define the following terms:

a. Division Algorithm theorem

b. Commutativity theorem (for propositions)

c. Conjunction semantic theorem

d. Contrapositive Proof theorem

Problem 3. Let $n \in \mathbb{N}$ be arbitrary. Prove that $n | n!$.

Problem 4. Let $a, b, c \in \mathbb{Z}$. Suppose that $a \leq b$. Prove that $a + c \leq b + c$.
Note: do not just cite a theorem.

Problem 5. Let p, q be propositions. Prove that $p \uparrow q \equiv \neg(p \wedge q)$.

Problem 6. Prove or disprove: $\forall x \in \mathbb{R}, x^2 \geq x$.

Problem 7. Prove or disprove: For arbitrary $x \in \mathbb{R}$, if x is irrational then $2x - 1$ is irrational.

Problem 8. Without using truth tables, prove the Composition Theorem:

$$(p \to q) \wedge (p \to r) \vdash p \to (q \wedge r).$$

Problem 9. State and prove modus tollens, using semantic theorems only (no truth tables).

Problem 10. Prove or disprove: $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}, |y| \leq |y + x|$.